

Probabilistic finite element prediction of knee wear simulator mechanics

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Abstract

Computational models have recently been developed to replicate experimental conditions present in the Stanmore knee wear simulator. These finite element (FE) models, which provide a virtual platform to evaluate total knee replacement (TKR) mechanics, were validated through comparisons with experimental data for a specific implant. As with any experiment, a small amount of variability is inherently present in component alignment, loading, and environmental conditions, but this variability has not been previously incorporated in the computational models. The objectives of the current research were to assess the impact of experimental variability on predicted TKR mechanics by determining the potential envelope of joint kinematics and contact mechanics present during wear simulator loading, and to evaluate the sensitivity of the joint mechanics to the experimental parameters. In this study, 8 component alignment and 4 experimental parameters were represented as distributions and used with probabilistic methods to assess the response of the system, including interaction effects. The probabilistic FE model evaluated two levels of parameter variability (with standard deviations of component alignment parameters up to 0.5 mm and 1°) and predicted a variability of up to 226% (3.44 mm) in resulting anterior–posterior (AP) translation, up to 169% (4.30°) in internal–external (IE) rotation, but less than 10% (1.66 MPa) in peak contact pressure. The critical alignment parameters were the tilt of the tibial insert and the IE rotational alignment of the femoral component. The observed variability in kinematics and, to a lesser extent, contact pressure, has the potential to impact wear observed experimentally.

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1. Introduction

Long-term performance of total knee replacement (TKR) components is influenced by joint kinematics and contact mechanics. The combination of contact stresses and relative motion contribute to wear and fatigue damage of the implant. In vitro studies have quantified the importance of kinematic conditions (Blunn et al., 1991; Kawanabe et al., 2001; McEwen et al., 2005; Schwenke et al., 2005) and contact pressure and area (Mazzucco and Spector, 2003; Sathasivam et al., 2001)

on wear. Kawanabe et al. (2001) measured a 6–11 fold increase in wear rates with the addition of internal–external (IE) rotation and anterior–posterior (AP) translation when compared to wear rates with only flexion and axial load profiles. A 50% reduction in kinematic inputs similarly resulted in a four-fold wear reduction in TKR simulator testing (McEwen et al., 2005).

Experimental testing with knee simulators and computational models (Fregly et al., 2005) are often used to quantitatively evaluate the performance of a specific design. The Stanmore knee wear simulator is forced-controlled and allows six-degree-of-freedom articulation of TKR implants during a simulated gait cycle (Walker et al., 1997; DesJardins et al., 2000). Recently, computational models representing the experimental Stanmore

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wear simulator conditions have been developed to efficiently evaluate implant design or surgical parameters (Godest et al., 2002; Halloran et al., 2005a). These models used explicit finite element (FE) methods to predict tibiofemoral kinematics and contact mechanics simultaneously. While these computational models have been verified by comparison with kinematic results from a single station of the wear simulator, effects of inherent variability in the experimental setup, such as scatter in implant positioning, have not been previously quantified. Thus, the objectives of this research were to assess the impact of experimental variability on predicted tibiofemoral mechanics by determining the potential envelope of joint kinematics and contact mechanics present during wear simulator loading, and to evaluate the sensitivity of the joint mechanics to the experimental parameters. It is hypothesized that the small variability in component alignment may lead to significant variability in TKR mechanics. Knowledge of the sensitivities allows identification of the critical experimental and numerical parameters to achieve consistent joint kinematics, contact pressures and ultimately, wear. In addition, understanding the sensitivities associated with an experimental gait loading configuration can lend insight into the important *in vivo* surgical placement parameters.

In order to predict the bounds of the tibiofemoral kinematics and contact mechanics resulting from simulated gait loading, variability in the experimental parameters is introduced into the computational model. This technique utilizing probabilistic methods results in a more rigorous validation of the computational model and a more realistic comparison of the predicted and experimental results. A probabilistic approach has advantages over traditional deterministic analyses in that the overall response to parameters and their variability is characterized, revealing interaction effects and the contributions of individual parameters. The traditional application of probabilistic methods is in the assessment of structural reliability (e.g. Melis et al., 1999; Kurth and Woods, 1992; Zhang and Liu, 2002), however, there have been several studies assessing the reliability of orthopaedic components (Nicolella et al., 2001; Browne et al., 1999; Dar et al., 2002). The approach used in this study uniquely applies probabilistic FE modeling to characterize and bound the potential variability in measured outcomes during an experimental test.

2. Methods

2.1. Deterministic FE model

The explicit FE model of the Stanmore knee wear simulator was previously validated by comparing

experimental kinematic measurements with model predictions (Halloran et al., 2005a). The FE model was developed from CAD models of a semi-constrained, fixed-bearing, cruciate-retaining TKR. The insert was represented with three-dimensional, 8-noded hexahedral elements (~8500), and rigid triangular surface elements (~19,000) were used for the femoral component. Through a previous convergence study, it was concluded that the mesh density utilized for these inserts was acceptable (Halloran et al., 2005a). In order to accomplish the hundreds of simulations necessary for a full probabilistic analysis, the femoral and tibial components were represented as rigid bodies, with a nonlinear pressure–overclosure relationship (Halloran et al., 2005b). The rigid body analysis requires approximately 6 min of analysis time for the gait cycle, and, with an optimized pressure–overclosure relationship specific to the implant and mesh utilized, has been shown to closely reproduce the kinematics and contact mechanics of a fully deformable analysis (Halloran et al., 2005b).

The loading conditions applied represented the force-controlled gait simulation (0.5 Hz) of the Stanmore knee simulator (Walker et al., 1997; ISO Standard 14243–2, 2000). The distal surface of the tibial insert was supported in the inferior–superior (IS) direction, and loading conditions applied to the insert included an AP load and IE torque. Varus–valgus (VV) and tilt of the insert were both constrained, and AP, medial–lateral (ML) and IE degrees of freedom (DOF) were free. The femoral component was constrained in IE, ML and AP DOF, unconstrained in VV and IS DOF, and flexion rotation was applied. The femoral axial load application was offset toward the medial condyle to reproduce the experimental 60–40% load split. Simulated soft-tissue constraint present in the knee simulator consists of a set of four springs that constrain the insert in AP displacement and IE rotation, and these were reproduced in the model (Fig. 1).

2.2. Probabilistic modeling methodology

Probabilistic methods were applied to the deterministic FE model to evaluate the impact of variability in the experimental parameters on several performance metrics. In this study, the probabilistic analysis software Nessus (SwRI, San Antonio, TX) was linked with the FE package Abaqus (Abaqus, Inc., Providence, RI) by custom scripting (Fig. 2). Using the input parameters represented as distributions, Nessus performed the variable perturbations. The modified FE model was run using Abaqus/Explicit and the simulated Stanmore gait loading conditions, and the resulting insert translations, rotations and contact mechanics were determined over the entire cycle.

Twelve experimental parameters (Table 1 and Fig. 1) dealing with component alignment, loading, and

environmental conditions were evaluated. The experimental parameters included four translations and four rotations of the femoral component and tibial insert. The rotations and translations defined the position of the femoral component and tibial insert relative to the fixed rotational axes (Fig. 1). In addition, 3 experimental set-up parameters (the spring constant, ML position of the spring constraint, and ML load split) and the coefficient of friction were also selected. The mean values were the deterministic values representing the neutral position of the implant in the Stanmore simulator. Each of the parameters was assumed to be independent and normally distributed. The standard deviations associated with the selected parameters were not available, nor easily measurable; as a result, two levels (A and B) were approximated from machining practice and engineering judgment (Table 1). The translational parameters were represented by their appropriate mean values with standard deviations of 0.25 mm (level A) and 0.5 mm (level B), while the

rotations had mean values of 0° with standard deviations of 0.5° and 1° for levels A and B, respectively. The mean and standard deviations for the experimental set-up parameters and friction (μ) were held fixed for the two levels (Table 1).

The probabilistic analysis implemented in this study used the advanced mean value (AMV) method, while Monte Carlo and AMV+Iteration methods were carried out to verify the accuracy and convergence of the response. A brief description of these methods is included; detailed description of these techniques can be found in Haldar and Mahadevan (2000) or Melchers (2001). The AMV family of methods is well suited for FE-based modeling because of its computational efficiency. The AMV methods combine optimization and reliability techniques to determine the most probable point (MPP), which represents the combination of parameter values that predict performance at the specified probability level. The AMV methods first transform the original variables into independent normal variables, use multi-variable optimization to locate the MPP, and then compute the performance value at the desired probability level. The AMV+Iteration method carries out additional evaluations beyond AMV to reach a specified level of convergence.

The result of the combined FE and probabilistic modeling was a bounded response over the gait cycle for the performance metrics: AP translation, IE rotation and contact pressure. In this study, AMV analyses were carried out for each performance metric at each percentile (1%, 50% and 99%) for each discrete location in the gait cycle. The AMV analysis was selected as a reasonable balance between accuracy and efficiency and the AMV results were validated by performing convergence studies with AMV+Iterations for several cases. A Monte Carlo analysis, which involves the random selection of parameters according to their distributions and subsequent evaluation of the performance metrics, was also carried out with 1000 trials for validation purposes.

The probabilistic sensitivity factors, α , are measures of how much the performance measure (e.g. AP

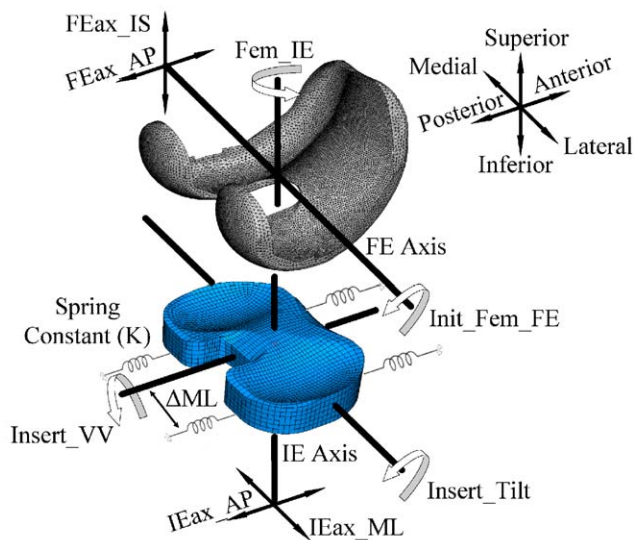


Fig. 1. Finite element model of TKR illustrating the study parameters (not shown: ML load split and μ).

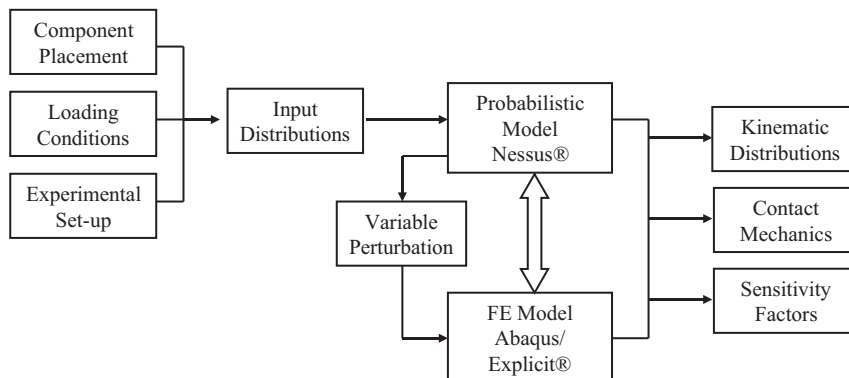


Fig. 2. Schematic of the probabilistic FE model.

Table 1
Study parameters with mean and standard deviation

Parameter	Description	Mean	Level A standard deviation	Level B standard deviation
FEax_AP	AP position of femoral FE axis	0 mm	0.25 mm	0.5 mm
FEax_IS	IS position of femoral FE axis	25.4 mm	0.25 mm	0.5 mm
IEax_AP	AP position of tibial IE axis	7.62 mm	0.25 mm	0.5 mm
IEax_ML	ML position of tibial IE axis	0 mm	0.25 mm	0.5 mm
Init_Fem_FE	Initial femoral FE rotation	0°	0.5°	1°
Fem_IE	Initial femoral IE rotation	0°	0.5°	1°
Insert_Tilt	Tilt (FE rotation) of the tibial insert	0°	0.5°	1°
Insert_VV	VV position of the tibial insert	0°	0.5°	1°
ΔML	ML position of spring fixation	28.7 mm	0.5 mm	0.5 mm
K	Spring constant	5.21 N/mm	0.09 N/mm	0.09 N/mm
ML_Load	ML load split (60%-40%)	60%	2.5%	2.5%
μ	Coefficient of friction	0.04	0.01	0.01

translation, IE rotation, contact pressure) is affected by each parameter. The sensitivities reported in this study are computed in the standard normal variate space and are relative indicators of the contributions of the variables. The sensitivity factors can change at different locations throughout the gait cycle, as certain parameters are more critical during stance than swing phase. In order to provide straightforward ranking of the variables, the absolute average of the sensitivity was calculated during the stance and swing phases and for the entire gait cycle for each of the performance measures.

3. Results

The predicted response envelopes utilizing the AMV, AMV+Iteration and Monte Carlo probabilistic methods had a maximum difference of 0.11 mm for AP translation and 0.22° for IE rotation. The good agreement of the results using the different probabilistic methods indicates an accurate and converged solution. As a result, all further results presented are based on the AMV analysis.

The predicted envelope of kinematic results for AP translation (Fig. 3) and IE rotation (Fig. 4) for the 2 variability levels are compared to experimental data. Data are presented at the 1 and 99 percentile levels for each metric over the entire gait cycle. Level A, with less variability, resulted in a narrower band than the Level B results. For AP translation (Fig. 3), the small deviations in experimental setup resulted in a maximum predicted range of 1.79 mm (Level A) and 3.44 mm (Level B) at 15% of the gait cycle. The maximum range in IE rotation was 2.17° (Level A) and 4.30° (Level B) at approximately 50% of the gait cycle. The maximum range of AP translation represented 226% of the corresponding experimental value, while the maximum range in IE represented 169% of the experimental value at the same temporal location. In addition, the predicted

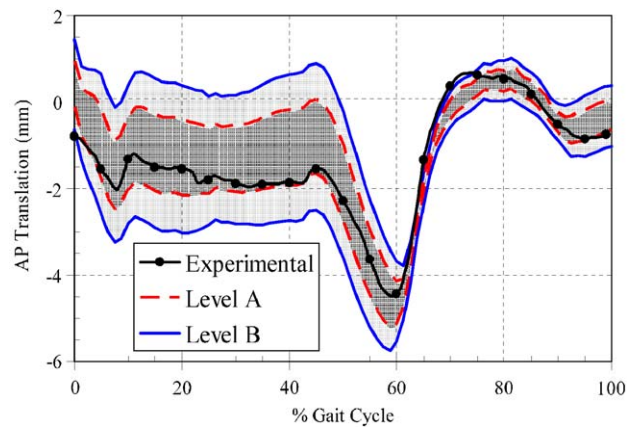


Fig. 3. Experimental AP translation (mm) with model-predicted envelope (1–99%) for variability levels A and B as a function of gait cycle.

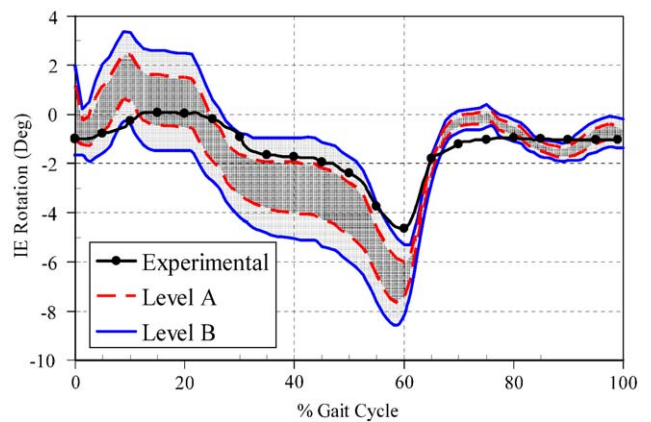


Fig. 4. Experimental IE rotation (°) with model-predicted envelope (1–99%) for variability levels A and B as a function of gait cycle.

distribution of peak contact pressures (Fig. 5) resulted in a maximum range of 1.66 MPa, representing approximately 10% of the mean peak contact pressure value of 16.92 MPa for Level B at 40% of the gait cycle.

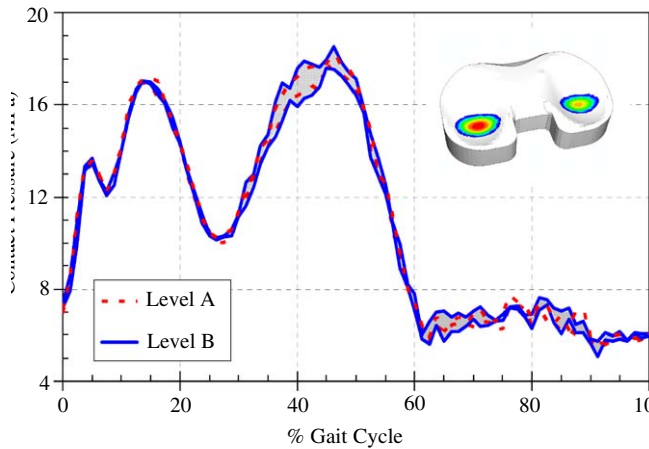


Fig. 5. Predicted envelope (1–99%) of peak contact pressure (MPa) for variability levels A and B as a function of gait cycle with typical contact patches.

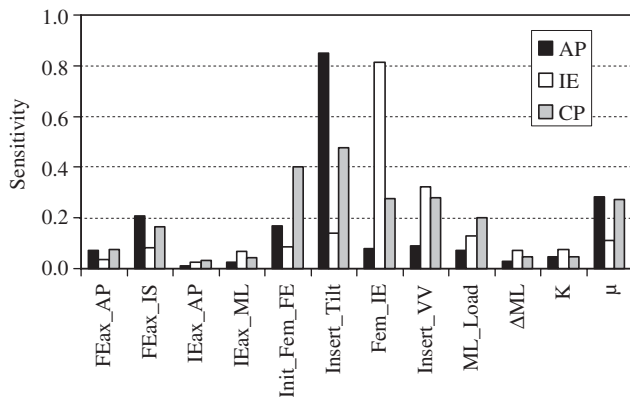


Fig. 6. Relative sensitivity results represented by the absolute average of the sensitivity over the gait cycle for AP translation, IE rotation and contact pressure.

Experimental data on contact pressure were not available for comparison. The largest variability in contact pressure occurred when the peak contact pressure was at its maximum in the stance phase. The computation time for a single trial was approximately 6 min on a ~3 GHz Intel computer; the AMV results for a single performance measure required approximately 24 h and 253 trials.

The impact each experimental parameter had on the kinematics or peak contact pressures varied significantly both between the parameters and throughout the gait cycle. The absolute averages of the sensitivities over the entire cycle presented in Fig. 6 illustrate the relative impact of the parameters on AP translation, IE rotation and contact pressure. Insert tilt was the greatest contributor to AP translation, while femoral IE alignment had the largest sensitivity factor for IE rotation. Unlike AP and IE where one single factor dominated the sensitivity factor results, peak contact pressure had 5 parameters with significant sensitivities, led by the tilt of

the insert and the femoral flexion alignment (Fig. 6); however, these sensitivity factors are not as meaningful due to the relatively small variability in the contact pressure data. An equally important finding of the sensitivity factor analysis is the identification of parameters that do not significantly affect performance. The spring constant K , the spring fixation distance (ΔML) and the location of the insert IE axis contributed negligibly to the three performance measures.

For each performance measure, the 5 most significant parameters (Fig. 7) were broken into their absolute averages over the stance (0–60% gait) and swing (60–100% gait) phases. The parameters with the greatest sensitivities, tilt of the insert for AP translation and femoral IE alignment for IE rotation, had larger sensitivities during the stance phase than the swing phase. With contact pressure, insert tilt was an important contributor during the stance phase, while tilt and femoral rotational alignment were equally important during the swing phase. Statistically significant differences between the parameters (Fig. 7) were evaluated using analysis of variance (ANOVA) with the Student–Newman–Keuls test (Hardeo and Ageed, 2000) and a level of significance $\alpha = 0.05$.

4. Discussion

In this study, a probabilistic FE model was developed to quantify the effect of experimental variability in a wear simulator by predicting the envelope of potential results for kinematic and contact mechanics measures. The Level B results, based on the larger standard deviations, indicated better agreement with the experimental kinematics than the Level A results for both AP translation and IE rotation. With IE rotation, it is noted that both levels overpredicted the peak rotation (60% gait cycle), which may be caused by the deterministic model not completely representing the inertial and frictional behavior of the simulator components.

The significant variability in kinematics (up to 226%) as a result of relatively small experimental deviations has the potential to impact the amount of wear observed experimentally, and underscores the need for careful experimental procedures. Kawanabe et al. (2001) and McEwen et al. (2005) have shown experimentally that changes in the magnitude of translation and/or rotation can substantially impact measured wear. McEwen et al. (2005) quantified a four-fold reduction in wear when the kinematics were scaled from 0–10 mm (AP) and $\pm 5^\circ$ (IE) to 0–5 mm and $\pm 2.5^\circ$. The observed variability ranges of up to 3.44 mm (AP) and 4.30° (IE) reported in this study therefore may lead to variability in wear patterns and weight loss. In contrast to the kinematics, it is unlikely that the contact pressure variability will significantly impact wear variability because of the small

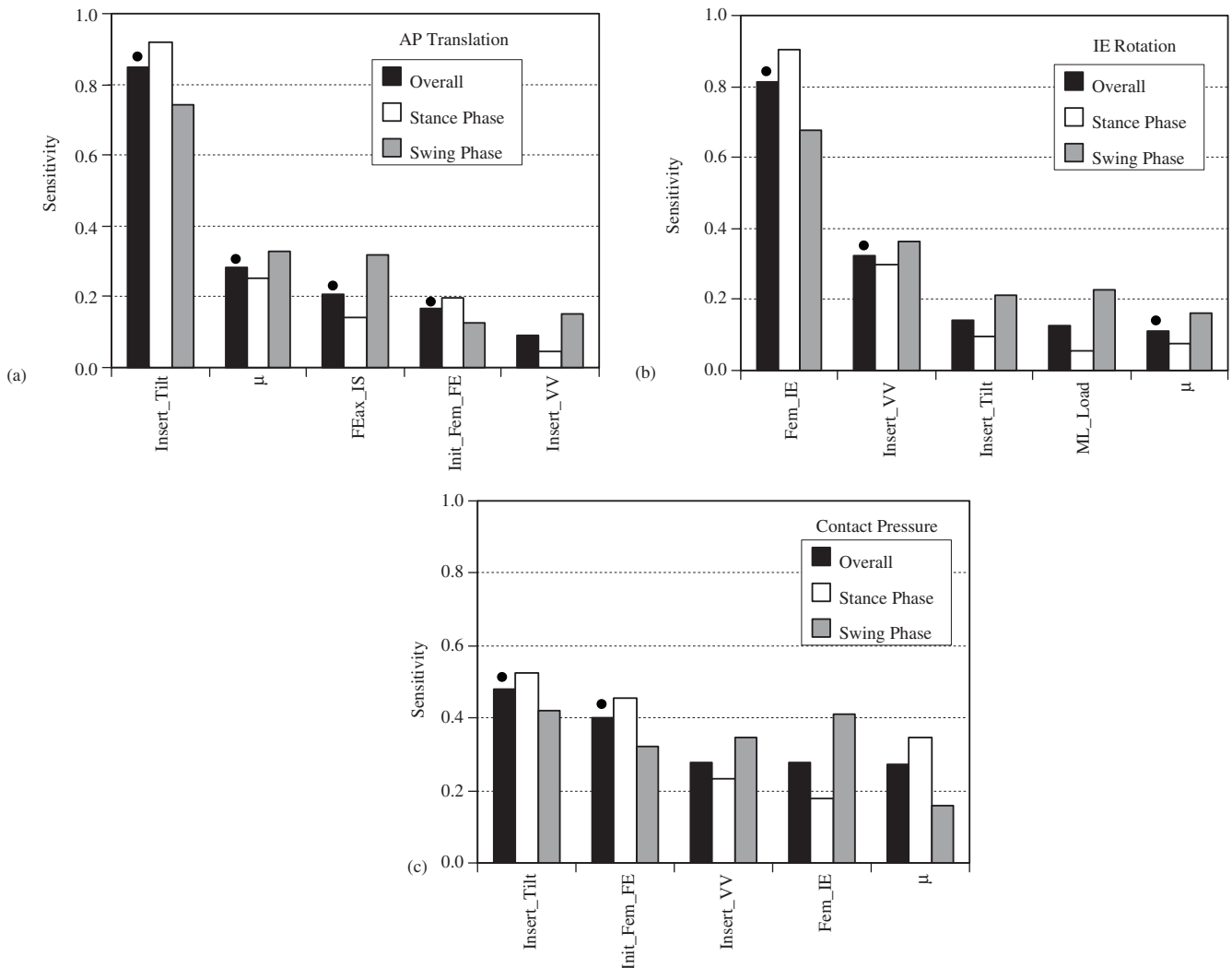


Fig. 7. Sensitivity of AP translation (a), IE rotation (b) and contact pressure (c) for the five most significant variables through the stance phase, swing phase and overall. • represents a statistically significant difference between sensitivity for the parameter and those below it ($\alpha = 0.05$).

range determined (less than 10%). In addition, although it is frequently assumed that increases in pressure will produce increases in wear, there is still controversy in the literature regarding the potential influence of pressure on TKR wear. For example, in pin-on-disk studies, Mazzucco and Spector (2003) concluded that wear was relatively independent of normal load, and hence contact pressure, over a 3.1–7.0 MPa range. In contrast, Sathasivam et al. (2001) used a pin-on-plate experiment with a pressure range of 2.9–23.8 MPa and found, in general, higher wear rates for higher pressures. Given the small pressure variability, the present results suggest that wear variability observed in the simulator is more likely due to changes in kinematics than pressure.

The sensitivity findings have identified the critical and non-critical variables in the experiment. The sensitivity factor results highlighted the significance of the rotational parameters, which tended to contribute more

significantly to the kinematic measures and contact pressure than the translational and experimental set-up parameters (Fig. 6). The greatest factor affecting both AP translation and peak contact pressure was tilt of the tibial insert, while the coefficient of friction was the largest of the secondary contributors. The femoral IE rotational alignment had the greatest effect on the IE kinematics, with the insert VV alignment as a secondary contributor. The factors found to be most significant (tilt and femoral IE alignment) are also commonly varied surgical parameters. Preserving the natural tibial tilt for the insert is frequently suggested to improve range of flexion motion (Walker and Garg, 1991; Dorr and Boiardo, 1986), although devices are also implanted with no tilt. In some cases, external rotation of the femoral component is suggested to improve patellofemoral tracking (Sodha et al., 2004; Miller et al., 2001). Results from this study suggest that variability in these

parameters will significantly influence the resulting joint mechanics. In addition, small variability associated with unintentional component placement and orientation may similarly impact joint mechanics in vivo.

The most critical assumption made in this study was the selection of the variability levels. Experimental data for these component placement and orientation parameters are not available, nor easily attainable. Yet, variability in these parameters is inherently present. A normal distribution with a standard deviation of 0.5 mm or 1° (Level B) implies that 95.5% of the values will be within ± 2 standard deviations (1 mm or 2°). Level A assumes half of this level of variability. The two levels were analyzed to demonstrate the significant impact that seemingly small variability can have on kinematics and contact mechanics. Notably, the assumed variability levels are considerably smaller than surgical alignment variability reported. In a study of 673 total knee arthroplasties, Mahaluxmivala et al. (2001) reported ranges of -8 to 13° in posterior tilt, 8 to -12° in insert varus–valgus and 8 to 22° in initial femoral flexion–extension. Similarly, Coull et al. (1999) measured 79 tibial varus–valgus angles and found a standard deviation of 2.84° and a range from 6 to -9° . Other limitations of this study included neglecting variability in the dimensions of the components themselves (tolerances) and in the simulator loading profiles. Lastly, the predictions are compared to kinematic results from only a single station of the wear simulator.

In this study, probabilistic methods provided an excellent framework for rigorous validation of a computational approach, as well as identification of the critical setup parameters. The probabilistic modeling demonstrated is a viable analysis tool largely due to the reduced computation times associated with explicit FE rigid body analysis with the optimized pressure–over-closure relationship. As computational models continue to become more realistic (e.g. soft tissue constraint, muscle loading), the probabilistic numerical framework developed in this study can be applied to quantify outcome-related performance measures and the critical placement parameters, thereby aiding in the realization of improved clinical outcomes.

In closing, a numerical tool was developed that combined probabilistic methods with explicit FE analysis in order to assess the impact of experimental variables on kinematics and contact mechanics in a knee wear simulator. The model evaluated the effect of variability in 12 parameters, including 4 translations and 4 rotations related to component alignment, on AP translation, IE rotation and peak contact pressure. The results were a predicted envelope bounding the 1 and 99 percentile results for each of the performance measures. The predicted bounds ranged significantly throughout the gait cycle; maximum ranges observed were 3.44 mm for AP translation, 4.30° for IE rotation and 1.66 MPa

for contact pressure for the parameter variability levels investigated (i.e. standard deviations of 0.5 mm and 1°). In addition, sensitivity factors showed that tilt of the tibial insert was the most critical parameter affecting AP translation and contact pressure, while initial IE rotational alignment of the femoral component was the most significant factor related to IE rotation. The methodology developed will be important in further analysis evaluating the robustness of TKR design to surgical and environmental variables.

Acknowledgement

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References

- Blunn, G.W., Walker, P.S., Joshi, A., Hardinge, K., 1991. The dominance of cyclic sliding in producing wear in total knee replacements. *Clinical Orthopaedics and Related Research* 273, 253–260.
- Browne, M., Langley, R.S., Gregson, P.J., 1999. Reliability theory for load bearing biomedical implants. *Biomaterials* 20, 1285–1292.
- Coull, R., Bankes, M.J.K., Rossouw, D.J., 1999. Evaluation of tibial component angles in 79 consecutive total knee arthroplasties. *The Knee* 6, 235–237.
- Dar, F.H., Meakin, J.R., Aspden, R.M., 2002. Statistical methods in finite element analysis. *Journal of Biomechanics* 35 (9), 1155–1161.
- DesJardins, J.D., Walker, P.S., Haider, H., Perry, J., 2000. The use of a force-controlled dynamic knee simulator to quantify the mechanical performance of total knee replacement designs during functional activity. *Journal of Biomechanics* 33, 1231–1242.
- Dorr, L.D., Boiardo, R.A., 1986. Technical considerations in total knee arthroplasty. *Clinical Orthopaedics and Related Research* 205, 5–11.
- Fregly, B.J., Sawyer, W.G., Harman, M.K., Banks, S.A., 2005. Computational wear prediction of a total knee replacement from in vivo kinematics. *Journal of Biomechanics* 38, 305–314.
- Godest, A.C., Beauginon, M., Haug, E., Taylor, M., Gregson, P.J., 2002. Simulation of a knee joint replacement during gait cycle using explicit finite element analysis. *Journal of Biomechanics* 35, 267–275.
- Haldar, A., Mahadevan, S., 2000. *Probability, Reliability and Statistical Methods in Engineering Design*. Wiley, New York, NY.
- Halloran, J.P., Petrella, A.J., Rullkoetter, P.J., 2005a. Explicit finite element modeling of total knee replacement mechanics. *Journal of Biomechanics* 38, 323–331.
- Halloran, J.P., Easley, S.K., Petrella, A.J., Rullkoetter, P.J., 2005b. Comparison of deformable and elastic foundation finite element simulations for predicting knee replacement mechanics. *Journal of Biomechanical Engineering* 127, 813–818.
- Hardeo, S., Ageed, M.I., 2000. *The Analysis of Variance: Fixed Random and Mixed Models*. Birkhäuser, Boston, MA.
- ISO Standard 14243-2, 2000. *Wear of total knee-joint prostheses, part 2: methods of measurement*. International Standards Organization.
- Kawanabe, K., Clarke, I.C., Tamura, J., Akagi, M., Good, V.D., Williams, P.A., Yamamoto, K., 2001. Effects of A–P translation and rotation on the wear of UHMWPE in a total knee joint simulator. *Journal of Biomedical Materials Research* 54, 400–406.

- Kurth, R.E., Woods, K.S., 1992. Probabilistic damage tolerant analysis for fatigue critical aircraft components. *ASME, Aerospace Division* 28, 89–97.
- Mahaluxmivala, J., Banks, M.J.K., Nicolai, P., Aldam, C.H., Allen, P.W., 2001. The effect of surgeon experience on component positioning in 673 press fit condylar posterior cruciate-sacrificing total knee arthroplasties. *Journal of Arthroplasty* 16 (5), 635–640.
- Mazzucco, D., Spector, M., 2003. Effects of contact area and stress on the volumetric wear of ultrahigh molecular weight polyethylene. *Wear* 254, 514–522.
- McEwen, H.M., Barnett, P.I., Bell, C.J., Farrar, R., Auger, D.D., Stone, M.H., Fisher, J., 2005. The influence of design, materials and kinematics on the in vitro wear of total knee replacements. *Journal of Biomechanics* 38, 357–365.
- Melchers, R.E., 2001. *Structural Reliability Analysis and Prediction*. Wiley, New York, NY.
- Melis, M.E., Zaretsky, E.V., August, R., 1999. Probabilistic analysis of aircraft gas turbine disk life and reliability. *Journal of Propulsion and Power* 15 (5), 658–666.
- Miller, M.C., Zhang, A.X., Petrella, A.J., Berger, R.A., Rubash, H.E., 2001. The effect of component placement on knee kinetics after arthroplasty with an unconstrained prosthesis. *Journal of Orthopaedic Research* 19, 614–620.
- Nicolella, D.P., Thacker, B.H., Katoozian, H., Davy, D.T., 2001. Probabilistic risk analysis of a cemented hip implant. *ASME Bioengineering Division* 50, 427–428.
- Sathasivam, S., Walker, P.S., Campbell, P.A., Rayner, K., 2001. The effect of contact area on wear in relation to fixed bearing and mobile bearing knee replacements. *Journal of Biomedical Materials Research (Applied Biomaterials)* 58, 282–290.
- Schwenke, T., Borgstede, L.L., Schneider, E., Andriacchi, T.P., Wimmer, M.A., 2005. The influence of slip velocity on wear of total knee arthroplasty. *Wear* 259, 926–932.
- Sodha, S., Kim, J., McGuire, K.J., Lonner, J.H., Lotke, P.A., 2004. Lateral retinacular release as a function of femoral component rotation in total knee arthroplasty. *Journal of Arthroplasty* 19, 459–463.
- Walker, P.S., Garg, A., 1991. Range of motion in total knee arthroplasty. *Clinical Orthopaedics and Related Research* 262, 227–235.
- Walker, P.S., Blunn, G.W., Broome, D.R., Perry, J., Watkin, S.A., Sathasivam, S., Dewar, M.E., Paul, J.P., 1997. A knee simulating machine for performance evaluation of total knee replacements. *Journal of Biomechanics* 30, 83–89.
- Zhang, Y., Liu, Q., 2002. Reliability-based design of automobile components. *Proceedings of the Institute of Mechanical Engineers, Part D. Journal of Automobile Engineering* 216 (6), 455–471.